LECTURE NOTE

ON

THEORY OF STRUCTURE (TH.2)

4TH SEMESTER IN CIVIL ENGG.

PREPARED BY

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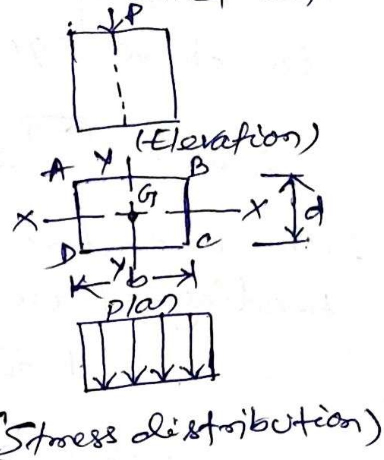
UNIT-I

Direct and Bending Stresses in vertical Members :-

Axial / direct Loads :- A load whose line of action coincides with the axis of a member is called an axial load or direct load.



cross-sectional area of column,

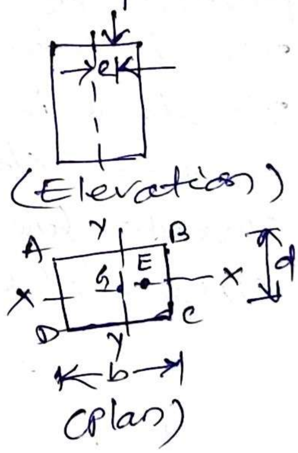
$A = b \times d$

$$\boxed{\text{Direct stress } \sigma_0 = \frac{P}{A} = \frac{P}{b \times d} \text{ (compressive)}}$$

Eccentric Loads :-

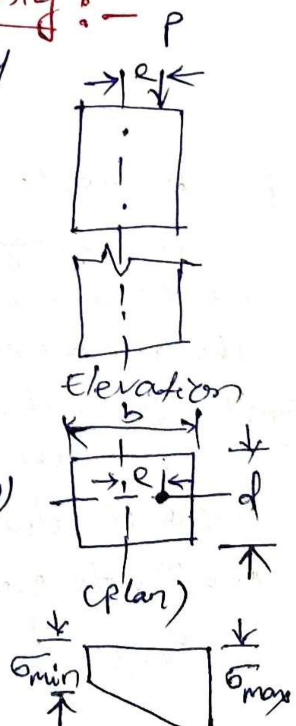
A load whose line of action does not coincide with the axis of member is called an eccentric load.

EX :- A bucket full of water, carried by a person in his hand.



Eccentricity About one Principal axis only :-

Let us a column member ABCD subjected to an eccentric load about one axis i.e, (about Y-Y axis).



Let P = Load acting on column

e = Eccentricity of load

b = width of column section

d = Thickness of the column

∴ Area of column section, $A = b \times d$

Moment of Inertia about centroidal axis (Y-Y)

$$I = \frac{db^3}{12}$$

and Section modulus, $Z = \frac{I}{y} = \frac{\frac{db^3}{12}}{b/2} = \frac{db^2}{6}$

We know that direct stress on the column due to the load $\sigma_0 = P/A$ and moment due to load $M = P \times e$

Bending stress at any point of column section at a distance y from y-y axis.

$$\sigma_b = \frac{My}{I} = \frac{M}{Z} \quad (\because Z = \frac{I}{y})$$

Now Bending stress at extreme, let us substitute $y = \frac{b}{2}$ in above equation. $\sigma_b = \frac{M \times b/2}{I} = \frac{M\frac{b}{2}}{\frac{db^3}{12}} = \frac{6M}{db^2} = \frac{6Pe}{db^2}$

$$\Rightarrow \sigma_b = \frac{6Pe}{Ab} \quad (\because I = \frac{db^3}{12},\ M = P \times e,\ A = b \times d)$$

$\therefore$ The eccentric load causes direct stress as well as bending stress. So the total stress at extreme fibre.

$$= \sigma_0 \pm \sigma_b = \frac{P}{A} \pm \frac{6Pe}{Ab} \quad \text{(in terms of eccentricity)}$$

$$= \frac{P}{A} \pm \frac{M}{Z} \quad \text{(in terms of modulus of section)}$$

The positive and negative sign will depend upon the position of fibre with respect to eccentric loading.
The stress is max^m at corner B & C (corner are near load)
The stress is min^m at corner A & D (corners are away from load)
The total stress along the width of column will vary by a straight line law.

The max^m stress, $\sigma_{max} = \frac{P}{A} + \frac{6P.e}{Ab} = \frac{P}{A}(1 + \frac{6e}{b})$ (in terms of eccentricity)

$$= \frac{P}{A} + \frac{M}{Z} \quad \text{(in terms of section modulus)}$$

and $\sigma_{min} = \frac{P}{A} - \frac{6Pe}{Ab} = \frac{P}{A}(1 - \frac{6e}{b})$ (in terms of eccentricity)

$$= \frac{P}{A} - \frac{M}{Z} \quad \text{(in terms of section modulus)}$$

P-1 A rectangular column 200mm wide and 150mm thick is carrying a vertical load of 120KN at an eccentricity of 50mm in a plane bisecting the thickness. Determine the maximum and minimum intensities of stress in the section.

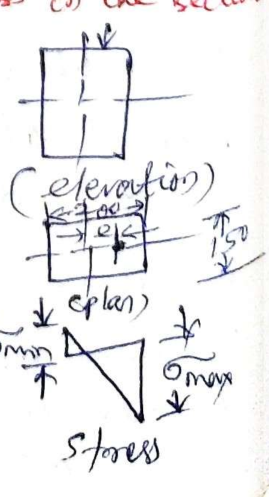
Sol^n Given that, $b = 200mm$, $d = 150mm$
$P = 120KN = 120 \times 10^3 N$, $e = 50mm$

$\therefore$ Area of column, $A = b \times d = 200 \times 150 = 30000\ mm^2$

Max^m intensity of stress in the section,

$$\sigma_{max} = \frac{P}{A}(1 + \frac{6e}{b}) = \frac{120 \times 10^3}{30000}(1 + \frac{6 \times 50}{200})\ N/mm^2$$

$$= 4(1 + 1.5) = 10\ N/mm^2 = 10\ MPa$$



Min^m intensity of stress in the section,

$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6e}{b}\right) = \frac{120\times10^3}{30000}\left(1-\frac{6\times50}{200}\right) N/mm^2$$

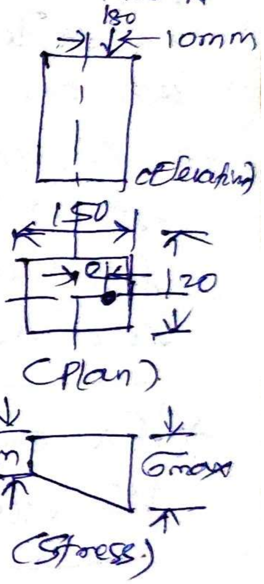
$$= \cancel{4(} = 4(1-1.5) = 4(-0.5) = -2 N/mm^2$$

$$= 2 N/mm^2 \text{ (tension)} = 2 MPa \text{ (tension)}$$

P-2 A rectangular strut is 150 mm and 120mm thick. It carries a load of 180 kN at an eccentricity of 10mm in a plane bisecting the thickness. Find the max^m & min^m intensities of stress in the section.

Sol^n Given that, $b = 1500mm$; $d = 120mm$, $P = 180 kN = 180\times10^3 N$

$e = 10mm$



Area of strut $A = b\times d = 150\times120 = 18000 mm^2$

max^m intensity of stress,

$$\sigma_{max} = \frac{P}{A}\left(1+\frac{6e}{b}\right) = \frac{180\times10^3}{18000}\left(1+\frac{6\times10}{150}\right) N/mm^2$$

$$\sigma_{max} = 10(1+0.4) = 14 N/mm^2 = 14 MPa$$

Min^m intensity of stress,

$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6e}{b}\right) = \frac{180\times10^3}{18000}\left(1-\frac{6\times10}{150}\right)$$

$$= 10(1-0.4) = 6 N/mm^2 = 6 MPa$$

Neutral Axis :- In resultant stress distribution diagram, when both compressive and tensile stresses are present, and in between these stresses, there is a layer of fibres which has neither compression nor tension. This layer is called the neutral layer or neutral surface. The intersection of this neutral surface with the axial plane of symmetry is call the neutral axis. At the neutral axis, the stress is zero.

Limit of Eccentricity :-

→ When an eccentric load is acting on a column it produces direct stress as well as bending stress.

→ On one side of Neutral axis there is a maximum stress (equal to sum of direct stress and bending stress) and on other side of neutral axis there is a minimum stress (equal to direct stress minus bending stress).

→ The bending stress remains less than the direct stress, the resultant stress is compressive.

→ If bending stress is equal to direct stress, then there will be a zero stress on one side, but if bending stress exceeds the direct stress, there there will be a stress on one side.

→ We saw that tensile stress is not to be permitted to come into play, then bending stress should be less than the direct stress, or, max^m, it may be equal to direct stress, i.e., $\sigma_b \leq \sigma_0$, $\frac{P \times e}{Z} \leq \frac{P}{A}$, $e \leq \frac{Z}{A}$

→ $\boxed{\text{for no tensile condition, the eccentricity, } e \leq \frac{Z}{A}}$

(a) Limit of eccentricity for a rectangular section;

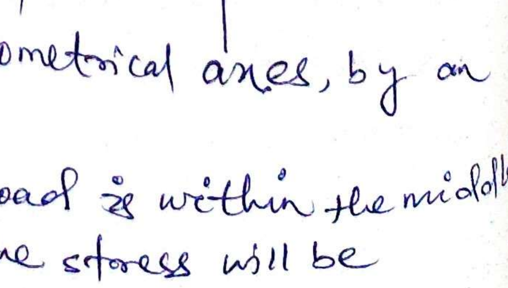
Section modulus, $Z = \frac{1}{6} bd^2$

Area, $A = b \times d$

No tension condition, $e \leq \frac{Z}{A}$

$\Rightarrow e \leq \frac{\frac{1}{6} bd^2}{bd}$

$e \leq \frac{1}{6} d$, It means the load



Can be eccentric on either side of geometrical axes, by an amount equal to $d/6$.

Thus if the line of action of the load is within the middle third, by the dotted area, then the stress will be compressive throughout.

(b) Limit of eccentricity for a hollow rectangular section:-

Consider a hollow rectangular section with B and D as outer width and thickness and b and d internal dimensions respectively, we know that the modulus of section,

$$Z = \frac{BD^3 - bd^3}{6D}$$ and area of hollow rectangular section.

$$A = BD - bd$$

We also know that for no tension condition,

$$e \leq \frac{Z}{A}$$

$$\leq \frac{\frac{BD^3 - bd^3}{6D}}{BD - bd}$$

$$\leq \frac{BD^3 - bd^3}{6D(BD - bd)}$$, it means that the load can be eccentric, on either side of the geometrical axis, by an amount equal to $\frac{BD^3 - bd^3}{6D(BD-bd)}$

(c) Limit of eccentricity of a circular section:-

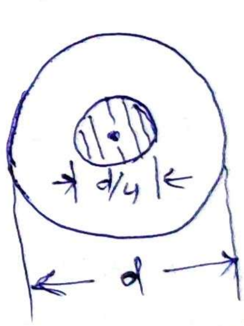
consider a circular section of diameter d as shown in figure. We know that the modulus of section,

$$Z = \frac{\pi}{32} \times d^3$$

and area of circular section,

$$A = \frac{\pi}{4} \times d^2$$

We also know that for no tension condition,



$$e \leq \frac{Z}{A}$$

$$\leq \frac{\pi/32\, d^2}{\frac{\pi}{4} d^2}$$

$$\leq d/8$$. It means that the load can be eccentric on any side of geometrical centre, by an amount equal to d/8. Thus, if the line of action of the load is within a circle of diameter equal to one-fourth of the main circle, then the stress will be compressive throughout.

(d) Limit of eccentricity for hollow circular section:-

Consider a hollow circular section of external and internal diameter as D and d respectively. We know that the modulus of section. $Z = \frac{\pi}{32} \times \frac{(D^4 - d^4)}{D}$ and area of hollow circular section, $A = \frac{\pi}{4}(D^2 - d^2)$

we also know that for no tension condition,

$$e \leq \frac{Z}{A} = \frac{\frac{\pi}{32}\frac{(D^4-d^4)}{D}}{\frac{\pi}{4} \times (D^2 - d^2)} = \frac{(D^2+d^2)}{8D} \quad \left(\because \frac{D^4 - d^4}{(D^2-d^2)} = (D^2+d^2)\right)$$

(b) Limit of eccentricity for a hollow rectangular section:-

Consider a hollow rectangular section with B and D as outer width and thickness and b and d internal dimensions respectively, we know that the modulus of section,

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$$\le \frac{\frac{BD^3 - bd^3}{6D}}{BD - bd}$$

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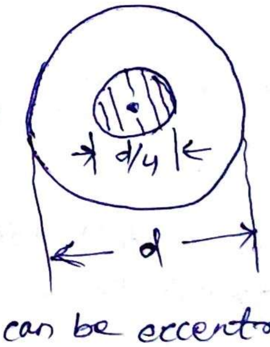

We also know that for no tension condition,

$$e \le \frac{Z}{A}$$

$$\le \frac{\pi/32 \, d^3}{\frac{\pi}{4} d^2}$$

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we also know that for no tension condition,

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It means that the load can be eccentric, on any side of the geometrical centre, by an amount equal to $\frac{D^2+d^2}{8D}$.

P-3 If a solid circular column of diameter 450mm is subjected to a load of 200kN at the outer edge, determine the max^m & min^m stresses in the section.

Sol^n- Since the load is applied at outer edge of circle, the eccentricity is the same at all the edges from the centroid of circular section. Therefore max^m stress (i.e. compressive) occurs at the location of load. and the min^m stress (i.e. tension) occurs at the opposite edge of circular section. Let eccentricity be considered with respect to Y-axis as shown in figure.

$$\sigma_{max} = \sigma_D = \frac{P}{A} + \frac{Pe}{Z_{YY}} \quad \text{and} \quad \sigma_{min} = \sigma_B = \frac{P}{A} - \frac{Pe_y}{Z_{YY}}$$

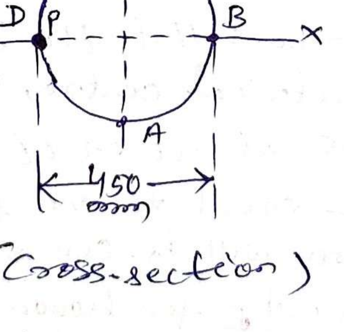
Here, $P = 200\,kN = 200 \times 10^3\,N$, $A = \frac{\pi D^2}{4} = \frac{\pi (450)^2}{4} = 159.043 \times 10^3\,mm^2$

$e_y = 225\,mm$, $Z_{YY} = \frac{\pi D^3}{32} = \frac{\pi \times 450^3}{32} = 8.946 \times 10^6\,mm^3$

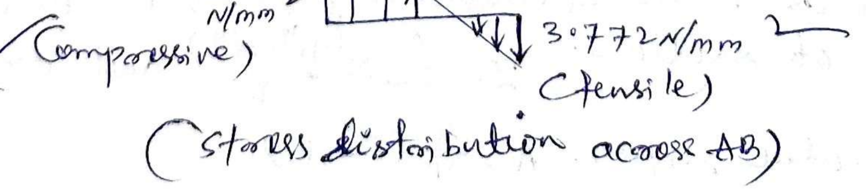
Stress at D, $\sigma_D = \frac{200 \times 10^3}{159.043 \times 10^3} + \frac{(200 \times 10^3) \times 225}{8.946 \times 10^6} = 1.258 + 5.030 = 6.288\,N/mm^2$ (Compression)

Stress at B, $\sigma_B = \frac{200 \times 10^3}{159.043 \times 10^3} - \frac{(200 \times 10^3) \times 225}{8.946 \times 10^6} = 1.258 - 5.030 = -3.772\,N/mm^2$ (tension)

Stress distribution across DB -



(Cross-section)



(Stress distribution across AB)

P-4 A column of 300X300 mm, made up of concrete with max^m allowable compressive stress of 25 N/mm^2 and max^m allowable tensile stress of 3.5 N/mm^2, find the greatest load that can be applied with an eccentricity limited to 75mm on the column.

Sol^n- Since the max^m & min^m stresses are limited to 25 N/mm^2 (compression) and 3.5 N/mm^2 (tension), respectively the appropriate formulas can be used as follows.

$$\sigma_{max} = \frac{P}{A} + \frac{P.e}{Z} = 25 \quad \text{and} \quad \sigma_{min} = \frac{P}{A} - \frac{P.e}{Z} = -3.5$$

from max^m stress equation,

$$\frac{P}{(300 \times 300)} + \frac{P \times 75}{\left(\frac{300 \times 300^2}{6}\right)} = 25 \Rightarrow \boxed{P = 900 \times 10^3 \text{ N}}$$

Similarly, from the minimum stress equation,

$$\frac{P}{(300 \times 300)} - \frac{P \times 75}{\left(\frac{300 \times 300^2}{6}\right)} = -3.5 \Rightarrow \boxed{P = 630 \times 10^3 \text{ N}}$$

when $P = 900 \times 10^3$ N is applied, the opposite extreme fibres will result in 25 N/mm^2 (comp.) and 5 N/mm^2 (tension) In this case even though the compressive stress is within the allowable vallue ($\sigma_{comp} \leq 25$ N/mm^2), the tensile stress (5 N/mm^2) exceeds the allowable value (i.e, $\sigma_{ten} \nleq 3.5$ N/mm^2) which is not desirable.

On the other hand, when $P = 630 \times 10^3$ N is applied, the opposite extreme fibres will result in 17.5 N/mm^2 (compression) and 3.5 N/mm^2 (tension), which satisfies the conditions. Therefore the greatest load can be applied safely is $P = 630 \times 10^3$ N.